

Cliffside Park Public Schools

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS) ALL STUDENTS WILL DEVELOP NUMBER SENSE AND WILL PERFORM STANDARD NUMERICAL OPERATIONS AND ESTIMATIONS ON ALL TYPES OF NUMBERS IN A VARIETY OF WAYS.	
Building upon knowledge and skills gained in preceding grades, by the end of Grade 8 , students will:	
4.1.8 A. Number Sense	
1. Extend understanding of the number system by constructing meanings for the following (unless otherwise noted, all indicators for grade 8 pertain to these sets of numbers as well):	
• Rational numbers	
• Percents	
• Exponents	
• Roots	
• Absolute values	
• Numbers represented in scientific notation	
2. Demonstrate a sense of the relative magnitudes of numbers.	
3. Understand and use ratios, rates, proportions, and percents (including percents greater than 100 and less than 1) in a variety of situations.	
4. Compare and order numbers of all named types.	
5. Use whole numbers, fractions, decimals, and percents to represent equivalent forms of the same number.	
6. Recognize that repeating decimals correspond to fractions and determine their fractional equivalents.	
• $5/7 = 0.714285714285\dots = 0.\overline{714285}$	
7. Construct meanings for common irrational numbers, such as π (pi) and the square root of 2.	
4.1.8 B. Numerical Operations	
1. Use and explain procedures for performing calculations involving addition, subtraction, multiplication, division, and exponentiation with integers and all number types named above with:	
• Pencil-and-paper	
• Mental math	
• Calculator	
2. Use exponentiation to find whole number powers of numbers.	

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3. Find square and cube roots of numbers and understand the inverse nature of powers and roots.	
4. Solve problems involving proportions and percents.	
5. Understand and apply the standard algebraic order of operations, including appropriate use of parentheses.	
4.1.8 C. Estimation	
1. Estimate square and cube roots of numbers.	
2. Use equivalent representations of numbers such as fractions, decimals, and percents to facilitate estimation.	
3. Recognize the limitations of estimation and assess the amount of error resulting from estimation.	
STANDARD 4.2 (GEOMETRY AND MEASUREMENT) ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE AND ANALYZE PHENOMENA.	
Building upon knowledge and skills gained in preceding grades, by the end of Grade 8 , students will:	
4.2.8 A. Geometric Properties	
1. Understand and apply concepts involving lines, angles, and planes.	
<ul style="list-style-type: none"> ● Complementary and supplementary angles 	
<ul style="list-style-type: none"> ● Vertical angles 	
<ul style="list-style-type: none"> ● Bisectors and perpendicular bisectors 	
<ul style="list-style-type: none"> ● Parallel, perpendicular, and intersecting planes 	
<ul style="list-style-type: none"> ● Intersection of plane with cube, cylinder, cone, and sphere 	
2. Understand and apply the Pythagorean theorem.	

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3. Understand and apply properties of polygons.	
<ul style="list-style-type: none">• Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi	
<ul style="list-style-type: none">• Regular polygons	
<ul style="list-style-type: none">• Sum of measures of interior angles of a polygon	
<ul style="list-style-type: none">• Which polygons can be used alone to generate a tessellation and why	
4. Understand and apply the concept of similarity.	
<ul style="list-style-type: none">• Using proportions to find missing measures	
<ul style="list-style-type: none">• Scale drawings	
<ul style="list-style-type: none">• Models of 3D objects	
5. Use logic and reasoning to make and support conjectures about geometric objects.	
6. Perform basic geometric constructions using a variety of methods (e.g., straightedge and compass, patty/tracing paper, or technology).	
<ul style="list-style-type: none">• Congruent angles or line segments	
<ul style="list-style-type: none">• Midpoint of a line segment	
7. Create two-dimensional representations (e.g., nets or projective views) for the surfaces of three-dimensional objects.	

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4.2.8 B. Transforming Shapes	
1. Understand and apply transformations.	
<ul style="list-style-type: none"> ● Finding the image, given the pre-image, and vice-versa 	
<ul style="list-style-type: none"> ● Sequence of transformations needed to map one figure onto another 	
<ul style="list-style-type: none"> ● Reflections, rotations, and translations result in images congruent to the pre-image 	
<ul style="list-style-type: none"> ● Dilations (stretching/shrinking) result in images similar to the pre-image 	
2. Use iterative procedures to generate geometric patterns.	
<ul style="list-style-type: none"> ● Fractals (e.g., the Koch Snowflake) 	
<ul style="list-style-type: none"> ● Self-similarity 	
<ul style="list-style-type: none"> ● Construction of initial stages 	
<ul style="list-style-type: none"> ● Patterns in successive stages (e.g., number of triangles in each stage of Sierpinski's Triangle) 	
4.2.8 C. Coordinate Geometry	
1. Use coordinates in four quadrants to represent geometric concepts.	
2. Use a coordinate grid to model and quantify transformations (e.g., translate right 4 units).	
4.2.8 D. Units of Measurement	
1. Solve problems requiring calculations that involve different units of measurement within a measurement system (e.g., 4'3" plus 7'10" equals	

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12 ² 1 ³).	
2. Use approximate equivalents between standard and metric systems to estimate measurements (e.g., 5 kilometers is about 3 miles).	
3. Recognize that the degree of precision needed in calculations depends on how the results will be used and the instruments used to generate the measurements.	
4. Select and use appropriate units and tools to measure quantities to the degree of precision needed in a particular problem-solving situation.	
5. Recognize that all measurements of continuous quantities are approximations.	
6. Solve problems that involve compound measurement units, such as speed (miles per hour), air pressure (pounds per square inch), and population density (persons per square mile).	
4.2.8 E. Measuring Geometric Objects	
1. Develop and apply strategies for finding perimeter and area.	
<ul style="list-style-type: none"> ● Geometric figures made by combining triangles, rectangles and circles or parts of circles 	
<ul style="list-style-type: none"> ● Estimation of area using grids of various sizes 	
<ul style="list-style-type: none"> ● Impact of a dilation on the perimeter and area of a 2-dimensional figure 	
2. Recognize that the volume of a pyramid or cone is one-third of the volume of the prism or cylinder with the same base and height (e.g., use rice to compare volumes of figures with same base and height).	
3. Develop and apply strategies and formulas for finding the surface area and volume of a three-dimensional figure.	
<ul style="list-style-type: none"> ● Volume - prism, cone, pyramid 	
<ul style="list-style-type: none"> ● Surface area - prism (triangular or rectangular base), pyramid (triangular or rectangular base) 	
<ul style="list-style-type: none"> ● Impact of a dilation on the surface area and volume of a three-dimensional figure 	

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4. Use formulas to find the volume and surface area of a sphere.	
STANDARD 4.3 (PATTERNS AND ALGEBRA) ALL STUDENTS WILL REPRESENT AND ANALYZE RELATIONSHIPS AMONG VARIABLE QUANTITIES AND SOLVE PROBLEMS INVOLVING PATTERNS, FUNCTIONS, AND ALGEBRAIC CONCEPTS AND PROCESSES.	
Building upon knowledge and skills gained in preceding grades, by the end of Grade 8 , students will:	
4.3.8 A. Patterns	
1. Recognize, describe, extend, and create patterns involving whole numbers, rational numbers, and integers.	
<ul style="list-style-type: none"> ● Descriptions using tables, verbal and symbolic rules, graphs, simple equations or expressions 	
<ul style="list-style-type: none"> ● Finite and infinite sequences 	
<ul style="list-style-type: none"> ● Arithmetic sequences (i.e., sequences generated by repeated addition of a fixed number, positive or negative) 	
<ul style="list-style-type: none"> ● Geometric sequences (i.e., sequences generated by repeated multiplication by a fixed positive ratio, greater than 1 or less than 1) 	
<ul style="list-style-type: none"> ● Generating sequences by using calculators to repeatedly apply a formula 	
4.3.8 B. Functions and Relationships	
1. Graph functions, and understand and describe their general behavior.	
<ul style="list-style-type: none"> ● Equations involving two variables 	
<ul style="list-style-type: none"> ● Rates of change (informal notion of slope) 	
2. Recognize and describe the difference between linear and exponential growth, using tables, graphs, and equations.	
4.3.8 C. Modeling	
1. Analyze functional relationships to explain how a change in one quantity	

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can result in a change in another, using pictures, graphs, charts, and equations.	
2. Use patterns, relations, symbolic algebra, and linear functions to model situations.	
<ul style="list-style-type: none"> ● Using concrete materials (manipulatives), tables, graphs, verbal rules, algebraic expressions/equations/inequalities 	
<ul style="list-style-type: none"> ● Growth situations, such as population growth and compound interest, using recursive (e.g., NOW-NEXT) formulas (cf. science standard 5.5 and social studies standard 6.6) 	
4.3.8 D. Procedures	
1. Use graphing techniques on a number line.	
<ul style="list-style-type: none"> ● Absolute value 	
<ul style="list-style-type: none"> ● Arithmetic operations represented by vectors (arrows) (e.g., “-3 + 6” is “left 3, right 6”) 	
2. Solve simple linear equations informally, graphically, and using formal algebraic methods.	
<ul style="list-style-type: none"> ● Multi-step, integer coefficients only (although answers may not be integers) 	
<ul style="list-style-type: none"> ● Simple literal equations (e.g., $A = lw$) 	
<ul style="list-style-type: none"> ● Using paper-and-pencil, calculators, graphing calculators, spreadsheets, and other technology 	
3. Solve simple linear inequalities.	
4. Create, evaluate, and simplify algebraic expressions involving variables.	
<ul style="list-style-type: none"> ● Order of operations, including appropriate use of parentheses 	
<ul style="list-style-type: none"> ● Distributive property 	

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<ul style="list-style-type: none"> ● Substitution of a number for a variable 	
<ul style="list-style-type: none"> ● Translation of a verbal phrase or sentence into an algebraic expression, equation, or inequality, and vice versa 	
<p>5. Understand and apply the properties of operations, numbers, equations, and inequalities.</p>	
<ul style="list-style-type: none"> ● Additive inverse 	
<ul style="list-style-type: none"> ● Multiplicative inverse 	
<ul style="list-style-type: none"> ● Addition and multiplication properties of equality 	
<ul style="list-style-type: none"> ● Addition and multiplication properties of inequalities 	
<p>STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS) ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.</p>	
<p>Building upon knowledge and skills gained in preceding grades, by the end of Grade 8, students will:</p>	
<p>4.4.8 A. Data Analysis</p>	
<p>1. Select and use appropriate representations for sets of data, and measures of central tendency (mean, median, and mode).</p>	
<ul style="list-style-type: none"> ● Type of display most appropriate for given data 	
<ul style="list-style-type: none"> ● Box-and-whisker plot, upper quartile, lower quartile 	
<ul style="list-style-type: none"> ● Scatter plot 	
<ul style="list-style-type: none"> ● Calculators and computer used to record and process information 	
<ul style="list-style-type: none"> ● Finding the median and mean (weighted average) using frequency data. 	

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<ul style="list-style-type: none"> ● Effect of additional data on measures of central tendency 	
2. Make inferences and formulate and evaluate arguments based on displays and analysis of data sets.	
3. Estimate lines of best fit and use them to interpolate within the range of the data.	
4. Use surveys and sampling techniques to generate data and draw conclusions about large groups.	
4.4.8 B. Probability	
1. Interpret probabilities as ratios, percents, and decimals.	
2. Determine probabilities of compound events.	
3. Explore the probabilities of conditional events (e.g., if there are seven marbles in a bag, three red and four green, what is the probability that two marbles picked from the bag, without replacement, are both red).	
4. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models.	
<ul style="list-style-type: none"> ● Frequency, relative frequency 	
5. Estimate probabilities and make predictions based on experimental and theoretical probabilities.	
6. Play and analyze probability-based games, and discuss the concepts of fairness and expected value.	
4.4.8 C. Discrete Mathematics—Systematic Listing and Counting	
1. Apply the multiplication principle of counting.	
<ul style="list-style-type: none"> ● Permutations: ordered situations with replacement (e.g., number of possible license plates) vs. ordered situations without replacement (e.g., number of possible slates of 3 class officers from a 23 student class) 	
<ul style="list-style-type: none"> ● Factorial notation 	

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<ul style="list-style-type: none"> ● Concept of combinations (e.g., number of possible delegations of 3 out of 23 students) 	
<p>2. Explore counting problems involving Venn diagrams with three attributes (e.g., there are 15, 20, and 25 students respectively in the chess club, the debating team, and the engineering society; how many different students belong to the three clubs if there are 6 students in chess and debating, 7 students in chess and engineering, 8 students in debating and engineering, and 2 students in all three?).</p>	
<p>3. Apply techniques of systematic listing, counting, and reasoning in a variety of different contexts.</p>	
<p>4.4.8 D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms</p>	
<p>1. Use vertex-edge graphs and algorithmic thinking to represent and find solutions to practical problems.</p>	
<ul style="list-style-type: none"> ● Finding the shortest network connecting specified sites 	
<ul style="list-style-type: none"> ● Finding a minimal route that includes every street (e.g., for trash pick-up) 	
<ul style="list-style-type: none"> ● Finding the shortest route on a map from one site to another 	
<ul style="list-style-type: none"> ● Finding the shortest circuit on a map that makes a tour of specified sites 	
<ul style="list-style-type: none"> ● Limitations of computers (e.g., the number of routes for a delivery truck visiting n sites is $n!$, so finding the shortest circuit by examining all circuits would overwhelm the capacity of any computer, now or in the future, even if n is less than 100) 	
<p>STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.</p>	
<p>At each grade level, with respect to content appropriate for that grade level, students will:</p>	
<p>4.5 A. Problem Solving</p>	
<p>1. Learn mathematics through problem solving, inquiry, and discovery.</p>	

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2. Solve problems that arise in mathematics and in other contexts.	
<ul style="list-style-type: none"> ● Open-ended problems 	
<ul style="list-style-type: none"> ● Non-routine problems 	
<ul style="list-style-type: none"> ● Problems with multiple solutions 	
<ul style="list-style-type: none"> ● Problems that can be solved in several ways 	
3. Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.	
4. Pose problems of various types and levels of difficulty.	
5. Monitor their progress and reflect on the process of their problem solving activity.	
6. Distinguish relevant from irrelevant information, and identify missing information.	
4.5 B. Communication	
1. Use communication to organize and clarify their mathematical thinking.	
<ul style="list-style-type: none"> ● Reading and writing 	
<ul style="list-style-type: none"> ● Discussion, listening, and questioning 	
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.	
3. Analyze and evaluate the mathematical thinking and strategies of others.	

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4. Use the language of mathematics to express mathematical ideas precisely.	
4.5 C. Connections	
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).	
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).	
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.	
4. Apply mathematics in practical situations and in other disciplines.	
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).	
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.	
4.5 D. Reasoning	
1. Recognize that mathematical facts, procedures, and claims must be justified.	
2. Use reasoning to support their mathematical conclusions and problem solutions.	
3. Select and use various types of reasoning and methods of proof.	
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.	
5. Make and investigate mathematical conjectures.	
<ul style="list-style-type: none"> • Counterexamples as a means of disproving conjectures 	

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<ul style="list-style-type: none"> ● Verifying conjectures using informal reasoning or proofs. 	
6. Evaluate examples of mathematical reasoning and determine whether they are valid.	
4.5 E. Representations	
1. Create and use representations to organize, record, and communicate mathematical ideas.	
<ul style="list-style-type: none"> ● Concrete representations (e.g., base-ten blocks or algebra tiles) 	
<ul style="list-style-type: none"> ● Pictorial representations (e.g., diagrams, charts, or tables) 	
<ul style="list-style-type: none"> ● Symbolic representations (e.g., a formula) 	
<ul style="list-style-type: none"> ● Graphical representations (e.g., a line graph) 	
2. Select, apply, and translate among mathematical representations to solve problems.	
3. Use representations to model and interpret physical, social, and mathematical phenomena.	
4.5 F. Technology	
1. Use technology to gather, analyze, and communicate mathematical information.	
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.	
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.	
4. Use calculators as problem-solving tools (e.g., to explore patterns, to	

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validate solutions).	
5. Use computer software to make and verify conjectures about geometric objects.	
6. Use computer-based laboratory technology for mathematical applications in the sciences (cf. science standards).	