



BOE Approved 8/18

**Cliffside Park Public Schools**

# **Cliffside Park Public Schools**

## **COURSE OF STUDY UNIT PLANNING GUIDE FOR: Grade 8 Unit 4**

Grade Level:  
8th

Cliffside Park School District  
Cliffside Park, NJ 07010  
[www.cliffsidepark.edu](http://www.cliffsidepark.edu)



**SUBJECT: MATHEMATICS MIDDLE SCHOOL**  
**BOE APPROVAL: August 2018**

**Cliffside Park Public Schools**

**GRADE: 8**

## Course Overview:

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

("Grade 8 » Introduction." *Grade 8» Introduction | Common Core State Standards Initiative*. Common Core State Standards Initiative, 2017. Web. 13 July 2017.)

### Overview of Units:

1. Real Numbers and Linear Equations
2. Using Functions to Model Relationships and Investigating BiVariate Data
3. Systems of Linear Equations and Congruence and Similarity
4. Geometry: Pythagorean Theorem and Finding Volume and Surface Area



## Mathematics: Grade 8

### Unit 2 Grade 8 Mathematics

**Unit Name:** Geometry: Pythagorean Theorem and Finding Volume and Surface Area (Topic 7, Topic 8)

**Primary Resource:** enVisionmath 2.0 Common Core 2017

**Topic Name:** **Topic 7:** Understanding and Applying the Pythagorean Theorem

**Topic Duration:** Approximately 2.5 weeks

**Unit Durations:** Approximately 6 weeks

#### TOPIC 7

##### Enduring Understandings:

- Many real-world problem situations can be represented with a mathematical model, but that model may not represent a real world, solution exactly.
- The Pythagorean Theorem can be used to determine if a triangle is a right triangle and to find the missing side length of a triangle.
- If a triangle has a length such that  $a^2 + b^2 = c^2$ , the triangle is a right triangle.
- The Pythagorean Theorem and its converse can be used to solve real-world problems that involve right triangles. Both can be used to determine the unknown leg lengths of a right triangle, or to identify or verify whether a triangle is a right triangle.
- The Pythagorean Theorem can be used to find the distance between any two points on a coordinate plane by drawing a line to connect the points as the hypotenuse of the right triangle where the leg are the horizontal and vertical distances.

#### Topic 7

##### Essential Question:

How can we use the Pythagorean Theorem to solve problems?



How does the Pythagorean Theorem relate the side lengths of a right triangle?  
 How can you determine if a triangle is a right triangle?  
 What kind of problems can be solved using the Pythagorean Theorem?  
 How can you use the Pythagorean Theorem to find the distance between two points?

**Focus of Standards:**

<b>Student Outcomes:</b> <i>What students will know.</i>	<b>Skills</b> <i>What students will be able to do.</i>	<b>Assessments</b>	<b>Resources</b>
<ul style="list-style-type: none"> <li>• Explain why double the base and the height of a triangle, the area is more than double.</li> <li>• Understand the Pythagorean Theorem.</li> <li>• Understand why the Converse of the Pythagorean Theorem is true.</li> <li>• Identify the coordinates of the third vertex of a triangle on the coordinate plane.</li> </ul>	<ul style="list-style-type: none"> <li>• Use mathematical modeling to represent a problem situation and to propose a solution.</li> <li>• Test and verify the appropriateness of their mathematical models.</li> <li>• Given two side lengths of a right triangle, use the Pythagorean Theorem to find the length of the third side.</li> <li>• Apply the Converse of the Pythagorean Theorem to analyze two-dimensional shapes.</li> </ul>	<p><b>Formative:</b></p> <ul style="list-style-type: none"> <li>• Diagnostic assessment</li> <li>• Topic Readiness Assessment</li> </ul> <p><b>Summative:</b></p> <ul style="list-style-type: none"> <li>• Topic Assessment</li> <li>• Topic Quiz</li> </ul> <p><b>Benchmark Tests:</b></p> <ul style="list-style-type: none"> <li>• Benchmark test given every 6-8 weeks.</li> </ul> <p><b>Alternative:</b></p> <ul style="list-style-type: none"> <li>• Topic Performance Task</li> <li>• Oral questioning</li> <li>• Journaling</li> </ul>	<p><b>Texts:</b></p> <p><i>enVision 2.0 Common Core</i></p> <p><b>Digital:</b></p> <ul style="list-style-type: none"> <li>• Student/Teacher eText</li> <li>• Videos</li> <li>• MathXL</li> <li>• IXL</li> <li>• 3-Act Mathematical Modeling</li> <li>• Virtual Nerd App</li> <li>• BouncePages App</li> <li>• Math Tools</li> </ul> <p><b>Classroom Math Materials</b></p> <ul style="list-style-type: none"> <li>• Student Journal</li> <li>• Online text</li> </ul>



	<ul style="list-style-type: none"> <li>• Apply the Pythagorean Theorem and its converse to solve real-world problems.</li> <li>• Apply the Pythagorean Theorem to find the distance between two points on a map or coordinate plane.</li> </ul>	<ul style="list-style-type: none"> <li>• Problems worked out partially</li> <li>• Using manipulatives to gauge understanding and develop reasoning skills</li> <li>• Using questioning strategies in TE.</li> <li>• Creating scaffolding questions on test</li> <li>• Online tests</li> <li>• Questions tied to Real-World scenarios</li> <li>• Projects</li> </ul>	<ul style="list-style-type: none"> <li>• Bounceapp</li> <li>• Digital toolkit</li> <li>• Math Practices and Problem Solving Handbook</li> </ul>
<b>VOCABULARY</b>			
<b>Tier 2</b> Leg, proof		<b>Tier 3</b> Hypotenuse, Pythagorean Theorem, CONverse of Pythagorean Theorem	
<b>NJSLS Math Standards</b>		<b>NJSLS Math Practices</b>	
<p><b>8.G.B.6b</b> Understand and apply the Pythagorean Theorem. Explain a proof of the Pythagorean Theorem and its converse.</p> <p><b>8.G.B.7</b> Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.</p> <p><b>8.G.B.8</b> Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>		<p><b>MP.1</b> Make sense of problems and persevere in solving them</p> <p><b>MP.2</b> Reason abstractly and quantitatively.</p> <p><b>MP.3</b> Construct viable arguments and critique the reasoning of others.</p> <p><b>MP.4</b> Model with mathematics.</p> <p><b>MP.5</b> Use appropriate tools strategically.</p>	



- MP.6** Attend to precision.
- MP.7** Look for and make use of structure.
- MP.8** Look for and express regularity in repeated reasoning.

**Unit Name:** Geometry: Pythagorean Theorem and Finding Volume and Surface Area and Final Project

**Primary Resource:** enVisionmath 2.0 Common Core 2017

**Topic Name:** **Topic 8:** Solving Problems Involving Surface Area and Volume

**Topic Duration:** Approximately 2.5 weeks and 1 week for Final Project

**Unit Durations:** Approximately 6 weeks

**TOPIC 8**

**Enduring Understandings:**

- Formulas for finding the areas of polygons, such as rectangles, squares, triangles as well as circles, can be used to find the surface areas of cylinders, cones and spheres.
- Finding the volume of a cylinder is an extension of finding the volume of a rectangular prism. The volume of a rectangular prism is the product of the area of its base and its height. Similarly, the volume of a cylinder is equal to the product of the area of its circular base and its height.
- The volume of a cone is  $\frac{1}{3}$  the volume of the cylinder given that the bases have the same radius and the heights are the same. The formula for the volume of a cone is  $V = \frac{1}{3}Bh$ , where  $B$  is the area of its circular base and  $h$  is the height of the cone.
- The volumes of a sphere and cone are proportionally related. The volume of a sphere is twice the volume of a cone that has the same circular base and height. The formula for the volume of a sphere is  $V = \frac{4}{3}\pi r^3$ , where  $r$  is the radius of the sphere.
- Many real-world problem situations can be represented with a mathematical model, but that model may not represent a real world, solution exactly.



- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of the math models.
- Explain why the results from their math models may not align to the problem situation.

**TOPIC 8**  
**Essential Questions:**

**Focus of Standards:**

<b>Student Outcomes:</b> <i>What students will know.</i>	<b>Skills</b> <i>What students will be able to do.</i>	<b>Assessments</b>	<b>Resources</b>
<ul style="list-style-type: none"> <li>• Recognize the relationship between the volume of a rectangular prism and the volume of a cylinder.</li> <li>• Recognize the relationship between the volume of a cylinder and the volume of a cone.</li> <li>• Recognize the relationship between the volume of a cone and the volume of a sphere.</li> </ul>	<ul style="list-style-type: none"> <li>• Find areas of cylinders, cones and spheres.</li> <li>• Solve real-world problems involving the volume of a cylinder.</li> <li>• Use the formula for volume of a cylinder to find an unknown measure.</li> <li>• Use the Pythagorean Theorem when solving volume problems.</li> <li>• Find the volume of a cone. Given the circumference of the base, find the volume of a cone.</li> <li>• Find the volume of a sphere. Given the surface area, find the volume of a sphere.</li> </ul>	<p><b>Formative:</b></p> <ul style="list-style-type: none"> <li>• Diagnostic assessment</li> <li>• Topic Readiness Assessment</li> </ul> <p><b>Summative:</b></p> <ul style="list-style-type: none"> <li>• Topic Assessment</li> <li>• Topic Quiz</li> </ul> <p><b>Benchmark Tests:</b></p> <ul style="list-style-type: none"> <li>• Benchmark test given every 6-8 weeks.</li> </ul>	<p><b>Texts:</b></p> <p><i>enVision 2.0 Common Core</i></p> <p><b>Digital:</b></p> <ul style="list-style-type: none"> <li>• Student/Teacher eText</li> <li>• Videos</li> <li>• MathXL</li> <li>• IXL</li> <li>• 3-Act Mathematical Modeling</li> <li>• Virtual Nerd App</li> <li>• BouncePages App</li> <li>• Math Tools</li> </ul> <p><b>Classroom Math Materials</b></p> <ul style="list-style-type: none"> <li>• Student Journal</li> <li>• Online text</li> </ul>



	<ul style="list-style-type: none"> <li>Find the volume of a composite figure.</li> </ul>	<p><b>Alternative:</b></p> <ul style="list-style-type: none"> <li>Topic Performance Task</li> <li>Oral questioning</li> <li>Journaling</li> <li>Problems worked out partially</li> <li>Using manipulatives to gauge understanding and develop reasoning skills</li> <li>Using questioning strategies in TE.</li> <li>Creating scaffolding questions on test</li> <li>Online tests</li> <li>Questions tied to Real-World scenarios</li> <li>Projects</li> </ul>	<ul style="list-style-type: none"> <li>Bounceapp</li> <li>Digital toolkit</li> <li>Math Practices and Problem Solving Handbook</li> </ul>
<b>VOCABULARY</b>			
<p><b>Tier 2</b> Cone, cylinder, sphere</p>		<p><b>Tier 3</b> Composite figure</p>	
<b>NJSLS Math Standards</b>		<b>NJSLS Math Practices</b>	
<p><b>8.G.C.9</b> Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. Know the formulas for the volumes</p>		<p><b>MP.1</b> Make sense of problems and persevere in solving them  <b>MP.2</b> Reason abstractly and quantitatively.</p>	





of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

**MP.3** Construct viable arguments and critique the reasoning of others.

**MP.4** Model with mathematics.

**MP.5** Use appropriate tools strategically.

**MP.6** Attend to precision.

**MP.7** Look for and make use of structure.

**MP.8** Look for and express regularity in repeated reasoning.

### Interdisciplinary Connections

NJSLS for ELA and Science are introduced, developed, and practiced in the context of learning math content and engaging in mathematical practices.

#### ELA

**ELA-Literacy.RST.6-8.3** Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks

**ELA-Literacy.RST.6-8.4.**Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics.

**ELA-Literacy.RST.6-8.3** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

**ELA-Literacy.SL.6.1**Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.

#### Science

**MS-ESS3-1** Construct a scientific explanation based on evidence for how the uneven distributions of Earth’s mineral, energy, and groundwater resources are the result of past and current geoscience processes.

**MS-ESS3-3** Apply scientific principles to design a method for monitoring and minimizing a human impact on the environment.\*



**MS-ETS1-1** Define the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles and potential impacts on people and the natural environment that may limit possible solutions.

**MS-ETS1-2** Evaluate competing design solutions using a systematic process to determine how well they meet the criteria and constraints of the problem.

**MS-ETS1-3.** Analyze data from tests to determine similarities and differences among several design solutions to identify the best characteristics of each that can be combined into a new solution to better meet the criteria for success.

**MS-ETS1-4.** Develop a model to generate data for iterative testing and modification of a proposed object, tool, or process such that an optimal design can be achieved.

### **NJSLS: 21st Century Life and Careers**

**Key Subjects and 21st Century:** Themes Mastery of key subjects and 21st century themes is essential to student success. Key subjects include English, reading or language arts, world languages, arts, mathematics, economics, science, geography, history, government and civics. In addition, schools must promote an understanding of academic content at much higher levels by weaving 21st century interdisciplinary themes into key subjects:

- Global Awareness
- Financial, Economic, Business and Entrepreneurial Literacy

**9.1.8.E.8** Compare the value of goods and services from different sellers when purchasing large quantities and small quantities.

**9.2.8.B.5** Analyze labor market trends using state and federal labor market information and other resources available online.

**9.1.8.B.7** Construct a budget to save for long-term, short-term, and charitable goals

**9.1.8.C.2** Compare and contrast credit cards and debit cards and the advantages and disadvantages of using each.

**9.1.8.C.5** Calculate the cost of borrowing various amounts of money using different types of credit (e.g., credit cards, installment loans, and mortgages) and compare the interest rates associated with each.

**Career Ready Practices:** Today's students need to develop thinking skills, content knowledge, and social and emotional competencies to navigate complex life and work environments.

**CRP2.** Apply appropriate academic and technical skills.

**CRP4.** Communicate clearly and effectively and with reason.

**CRP8.** Utilize critical thinking to make sense of problems and persevere in solving them.

**CRP11.** Use technology to enhance productivity



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## NJSLS Technology Standards

**8.1.8.A.4** Graph and calculate data within a spreadsheet and present a summary of the results

**8.1.8.C.1** Collaborate to develop and publish work that provides perspectives in a global problem for discussions with learners from other countries

**8.1.8.F.1** Explore a local issue, by using digital tools to collect and analyze data to identify and make an informed decision.

## Additional Resources:

[www.ixl.com](http://www.ixl.com)

[Khan Academy Grade 7](#)

[Illustrative Mathematics Grade 7](#)



<b>Integrated Differentiation/Accommodations/Modifications for Math 8 Unit 4</b> <i>(Alternate Modes of Instruction and Support)</i>		
<b>Modifications to Support Gifted and Talented Students</b>	<b>Modifications to Support English Language Learners</b>	<b>Modifications to Support Our Learners (Students with IEPs/504s and At-Risk Learners)</b>
<p>Integrate Higher Order Thinking Skills (HOTS) through questioning and extension projects specific to rational numbers, additive inverse, and properties of rational numbers.</p> <p>Provide menu of challenge activities for when the child finishes the lesson early (integrate technology when possible).</p> <p>College/Career Readiness skill enhancement - G &amp; T students can research professions related to the Rational Numbers.</p> <p>Have the student teach the lesson - peer tutoring (research-based strategy) Accelerate pace for students who are advanced in concepts.</p> <p>Use inquiry-based, discovery learning approaches that emphasize open-ended problems with solving problems with Rational Numbers.</p> <p>Allow students to design their own ways to find the answers to complex questions.</p>	<p><b>Concept/Idea Map</b> - teacher models note-taking on the modeling associated with using the Pythagorean Theorem and its converse and finding area and volumes of 2-dimensional and 3-dimensional shapes.</p> <p><b>Contextualize language</b> for the following key vocabulary terms: <b>Leg, proof, Hypotenuse, Pythagorean Theorem, Converse of Pythagorean Theorem, Cone, cylinder, sphere, and composite figure.</b></p> <p><b>Visuals and illustrations</b> Using a number line to add and subtract integers. Using integer tiles to represent positive and negative numbers. Using Fraction Tiles to represent adding, subtracting, multiplying and dividing fractions.</p> <p><b>Word/picture bank</b> available for students' reference; as well as Spanish translation through Google or their textbooks.</p>	<p>Review student individual educational plan and/or 504 plan for instructional, assessment, and environmental supports.</p> <p>Allow student to use calculator to simplify algebraic expressions with rational numbers.</p> <p>Teach students how to check the accuracy of the solution that was derived from use of a calculator.</p> <p>Provide manipulatives to aid in operations with rational numbers. (integer tiles and fraction tiles).</p> <p>Teach students how to check the accuracy of the solution that was derived from use of the calculator or other method.</p> <p>Provide number lines to help students with adding or subtracting integers.</p> <p>Use integer tiles to perform operations with rational numbers.</p> <p>Utilize graphic organizer or partially completed template for students to simplify expressions..</p>



**Wait Time Two** - extend basic "Wait Time" - after the 1st student responds to a question, the teacher waits an additional 5 - 7 seconds before calling on another student to ask a question about scatter plots, correlations and lines of best fit;

**Native Language Supports:** Working with peer, online assistive technology, translation device, bilingual dictionary.

Teach the text backward - **frontload** the concepts and vocabulary needed for learning the material and activating prior knowledge about absolute values, properties of mathematics and integers.

Use a **word square** to teach target academic vocabulary for the unit.

Provide study guides that are partially completed by teacher, allowing the student to fill in missing information during instruction in order to aid in obtaining information pertaining to modeling linear functions and solving linear systems.

Utilize visual aids such as charts or graphs connected to rational numbers.

By utilizing individual student assessment results, the teacher will provide small group or remedial instruction to review essential questions/big ideas of rational numbers, properties of numbers, operations with rational numbers, and operations with fractions.

Provide wait time to allow students to process orally presented information and questions relating to each topic.

Access to word/picture banks to develop an understanding and use content-specific vocabulary, such as those listed under **Contextualize language**.

Allow for Student Choice: Students should be permitted to demonstrate understanding of content through illustrations, computer projects, oral response, creative presentations or demonstration, etc.

Support comprehension of unknown vocabulary, by providing examples of Note-taking, highlighting, underlining, etc. Students should be allowed given copies of grade level material or text so they can highlight or underline pertinent information.



		<p>Text to speech/Oral reading</p> <p>Provide students with flexible seating options while working independently, depending on need or preference.</p> <p>Math-specific vocabulary and literary terms should be pre-taught before teaching relevant concepts.</p> <p>Allow extra time to complete in class written assignments.</p> <p>Provide students with a sample problem or list of steps or procedures for multi-step solutions to problems. Allow student to reference these procedures when solving independently.</p> <p>Reduce the number of assigned problems within each topic..</p> <p>Provide models or templates to teach the structure of how to solve problems systematically.</p> <p>If necessary, provide additional set of materials or online access so that students can utilize resources at school and home.</p> <p>Provide study guide for students to review before Unit 1 quizzes and tests.</p> <p>Modify tests to address big ideas/essential questions of the unit..</p> <p>Provide instructional adaptations and interventions in the general education classroom.</p> <ul style="list-style-type: none"><li>• Differentiated Intervention (enVision 2.0)</li></ul>
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		<ul style="list-style-type: none"> <li>○ Reteach</li> <li>○ Additional Vocabulary Support</li> <li>○ Build Mathematical Literacy</li> <li>○ Math tools and Games</li> <li>● MATHXL</li> <li>● IXL</li> </ul> <p>Intensive individual intervention:</p> <ul style="list-style-type: none"> <li>● Rtl in enVision 2.0</li> </ul>
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**Sources**

New Jersey Student Learning Standards (2016) <http://www.state.nj.us/education/cccs/2016/math/standards.pdf>  
 New Jersey Student Learning Standards: Technology (2014) - <http://www.state.nj.us/education/cccs/2014/tech/8.pdf>  
 New Jersey Student Learning Standards: ELA (2014) - <https://www.state.nj.us/education/cccs/2016/ela/g03.pdf>  
 New Jersey Science and Engineering Practices - <https://www.state.nj.us/education/cccs/2016/science/>  
 NJ Career Ready Practices (2014) - <https://www.state.nj.us/education/cccs/2014/career/CareerReadyPractices.pdf>  
 Pearson enVision 2.0 (2016) <https://www.pearsonrealize.com/index.html#/>